MAHARASHTRA AGRICULTURAL UNIVERSITIES EXAMINATION BOARD, PUNE
SEMESTER END EXAMINATION

B.Sc. (Agri.) / B.Sc. (Hort.) / B.Sc. (Forestry) / B.B.A. (Agri.) / B.Sc. (Agril. Bio-Tech.)

Semester	: I (Old)	**Term**	: I	**Academic Year**	: 2017-18
Course No.	: MATH 111	**Title**	: Mathematics (Deficiency Course)		
Credits	: 2(1+1)				
Day & Date	: Tuesday, 26.12.2017	**Time**	: 14.00 to 17.00	**Total Marks**	: 80

Note :
1. Solve **ANY EIGHT** questions from **SECTION "A"**.
2. All questions from **SECTION "B"** are compulsory.
3. All questions carry equal marks.
4. Draw neat diagrams wherever necessary.

SECTION "A"

Q.1 a) Prove that the sum of roots of quadratic equation $ax^2+bx+c = 0$ is $-b/a$ where $a \neq 0$.

b) Form the quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$

Q.2 a) Apply Simpson's rule to find the area of a plot having the following dimensions. Ordinates 4, 7, 8, 10, 7, 6, 3 meters and common distance 1 meter.

b) Find the distance between the two points A (4, 0) and B (0, 3).

Q.3 a) State any four properties of determinant with example.

b) Evaluate the following determinant.

$$\begin{vmatrix} 3 & 0 & 2 \\ 1 & -1 & 1 \\ 4 & 3 & 3 \end{vmatrix}$$

Q.4 a) Define logarithm. Show that $\log_a(m.n) = \log_a m + \log_a n$.

b) Evaluate (i) $\log_{343} 7$ (ii) $\log_5 625$

Q.5 a) Find the equation of a circle having its center at the point (2,0) and radius unit.

b) Find the co-ordinates of the centre and radius of a circle whose equation is $x^2+y^2+6x-8y=0$.

Q.6 a) State any four theorems of limit.

b) Evaluate the following limits (Any Two).

1) $\lim_{x\to 1}(y+1)(y-1)$ 2) $\lim_{x\to 2}\frac{x^3-8}{x^5-32}$ 3) $\lim_{x\to 0}\frac{\sin 3x}{x}$

Q.7 a) A vertical flagstaff stands on a horizontal plane. The angle of elevation of its top was found to be 30^0 from a point at a distance of 52.5m from its foot. Find the height of the flagstaff.

b) Enlist any four types of functions with one example each.

Q.8 a) Find the equation of straight line passing through the points whose co-ordinates are (-1, 3) and (6, -7).

b) Find the co-ordinates of the points which externally divides the line joining the points P (-2, 1) and Q(5,7) in the ratio 2:1.

(P.T.O)

Q.9 Differentiate the following functions w. r. t 'x' (Any Two).

a) $2x^3 - 3x^2$ b) $x^2 \sin x$ c) $(3x^2-5x+7)^{10}$

Q.10 Evaluate the following integrals (Any Four).

a) $\int \frac{x^3 - 3x^2 + 5x - 2}{x^3} dx$ b) $\int_a^b e^x dx$ c) $\int \sin 2x \, dx$

d) $\int_0^2 (3x^2 - 2x) \, dx$ e) $\int_0^2 (1/x) \, dx$

SECTION "B"

Q.11 Fill in the blanks.

1) A quadratic equation cannot have more than __________ roots.
2) If any two rows or columns of the determinant are identical, then the value of the determinant is __________.
3) $\log_5 5$ = __________.
4) If a point lies in the third quadrant then both the co-ordinates of a point are______.
5) y = log x +10 is __________type of function.
6) The radius of the circle $x^2+y^2=16$ is __________.
7) $\int a^2 \, dx$ = __________.
8) Limit k = __________, where k is constant.

Q.12 State True or False.

1) The root of quadratic equation $ax^2+bx+c = 0$, where $a \neq 0$ will be real and equal if $b^2-4ac = 0$.
2) The definite integral of a function is unique.
3) Simpson's rule is used when even number of ordinates is given.
4) log (mn) = logm - logn.
5) Derivative of constant function is zero.
6) The equation of y axis is x = 0.
7) Point (1, -1) lies in the first quadrant.
8) e^x is a logarithmic function.

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EXAMINATION BOARD -PUNE
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B.Sc. (Agri)

Semester: I (Old)	Academic year: 2017-18
Course No.: MATH-111	Course Title: Mathematics (Deficiency Course)
Credits : 2(1+1)	Total Marks: 80
Day and Date:-	Time:

MODEL ANSWER

Note: 1) Solve Any Eight questions from SECTION "A".
2) All questions from SECTION "B" are compulsory.
3) All questions carry equal marks.
4) Draws neat diagram wherever necessary.

SECTION "A"

Q.1 (a) Prove that the sum of roots of quadratic equation $ax^2+bx+c=0$ is $-b/a$ where $a \neq 0$.

Solution: Given that $ax^2 + bx + c = 0, a \neq 0$

Let α and β are two roots of this equation

i.e. $\alpha = \dfrac{-b+\sqrt{b^2-4ac}}{2a}$ and $\beta = \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

Adding α and β gives

$$\alpha + \beta = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{(-b+\sqrt{b2-4ac}) + (-b-\sqrt{b2-4ac})}{2a}$$

$$= \frac{-2b}{2a} = \frac{-b}{a}$$

$$\text{ie } \alpha + \beta = \frac{-b}{a} = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(b) Form the quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

$\alpha = 2+\sqrt{3}$ $\beta = 2-\sqrt{3}$

$(x-\alpha)(x-\beta) = 0$

$[x-(2+\sqrt{3})][x-(2-\sqrt{3})] = 0$

$(x-2-\sqrt{3})(x-2+\sqrt{3}) = 0$

$x^2 - 2x + \sqrt{3}x - 2x + 4 - 2\sqrt{3} - \sqrt{3}x + 2\sqrt{3} - 3 = 0$

$x^2 - 4x + 1 = 0$

Q.2 (a) Apply the Simpson's rule to find the area of a plot having the following dimensions. Ordinates 4,7,8,10,7,6,3 meters and common distance 1 meter.

Ans.- Given ordinates $P_1 = 4, P_2 = 7, P_3 = 8, P_4 = 10, P_5 = 7, P_6 = 6, P_7 = 3$

Common distance d = 1 meter

Therefore by using Simpson's rule

$$\text{Area} = \frac{d}{3}[(P_1 + P_7) + 2(P_3 + P_5) + 4(P_2 + P_4 + P_6)]$$

$= \frac{1}{3}[(4+3)+2(8+7)+4(7+10+6)]$

$= \frac{1}{3}[7+2(15)+4(23)]$

$= \frac{1}{3}[7+30+92]$

$= \frac{129}{3}$

$=$ 43 Sq. m.

(b) Find the distance between the two points A(4,0) and B(0,3).

Ans. – let $A(X_1, Y_1) = A(4, 0)$

$B(X_2, Y_2) = B(0, 3)$

By using distance formula

$\ell(AB) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-4)^2 + (3-0)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$ unit

Q.3 (a) State any four properties of determinant with example.

Ans. (1) The value of the determinant is not changed (altered) by interchanging rows and columns in same order.

e.g. $\begin{vmatrix} 2 & 6 \\ 5 & 7 \end{vmatrix} = 14 - 30 = -16$

By changing rows & columns,

e.g. $\begin{vmatrix} 2 & 5 \\ 6 & 7 \end{vmatrix} = 14 - 30 = -16$

(2) If any two rows and two columns of determinant interchanged, then the value of determinant changes by sign only

e.g. $\begin{vmatrix} 2 & 3 & 5 \\ 6 & 0 & 4 \\ 1 & 3 & 9 \end{vmatrix} = 2(0-12) - 3(54-4) + 5(18-0)$
$= -24 - 150 + 90 = -84$

By changing 2^{nd} & 3^{rd} row

e.g. $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 3 & 9 \\ 6 & 0 & 4 \end{vmatrix} = 2(12-0) - 3(4-54) + 5(0-18)$
$= 24 + 150 - 90 = 84$

(3) If the rows or columns of determinant are same (identical) the value of determinant is zero.

e.g. $\begin{vmatrix} 2 & 3 & 5 \\ 1 & 3 & 9 \\ 1 & 3 & 9 \end{vmatrix} = 2(27-27) - 3(9-9) + 5(3-3)$
$= 0 - 0 + 0 = 0$

(4) If all the elements of one row (or column) are multiplied by same number 'K' the value of determinant is 'K' times the value of given determinant (or we can take the 'K' common from that row or column).

e.g. $\begin{vmatrix} 2k & 5k \\ 6 & 7 \end{vmatrix} = k \begin{vmatrix} 2 & 5 \\ 6 & 7 \end{vmatrix} = k(14-30) = k(-16) - 16k$

6

6/4

(5) If each element of one row (or column) of determinant consists of sum of the terms or parts then determinant can be written as the sum of the two determinants.

e.g. $\begin{vmatrix} 2+3 & 3 & 5 \\ 1+2 & 3 & 9 \\ 6+7 & 0 & 4 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 5 \\ 1 & 3 & 9 \\ 6 & 0 & 4 \end{vmatrix} + \begin{vmatrix} 3 & 3 & 5 \\ 2 & 3 & 9 \\ 7 & 0 & 4 \end{vmatrix}$

(6) If all the elements of one row or (column) are zero, the value of determinant is zero.

e.g. $\begin{vmatrix} 2 & 3 & 5 \\ 0 & 0 & 0 \\ 6 & 0 & 4 \end{vmatrix} = 2(0-0) - 3\,(0-0) + 5(0-0) = 0 - 0 + 0 = 0$

(b) Evaluate the following determinant.

$$\begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & 3 \end{vmatrix} = 3\begin{vmatrix} -1 & 1 \\ 3 & 3 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 4 & 3 \end{vmatrix} + 2\begin{vmatrix} 1 & -1 \\ 4 & 3 \end{vmatrix}$$

$= 3(-3 -3) - 0\,(3-4) + 2\,(3-(-4))$
$= 3(-6) - 0 + 2(7)$
$= -18 + 14$
$= -4$

Q.4 (a) Define logarithm. Show that $\log_a(m.n) = \log_a m + \log_a n$.

Ans. - Definition: the logarithm of any number to a given base is the index of the power to which the base must be raised in order to equal the given number.

Let $x = \log_a m$ and $y = \log_a n$
Here $m = a^x$ and $n = a^y$
Thus $m.\,n = a^x.\,a^y$
$m.\,n = a^{x+y}$
$x + y = \log_a m\,n$
Thus $\log_a mn = \log_a m + \log_a n$

(b) Evaluate (i) $\log_{343}7$

Let $\log_{343}7 = x$
$343^x = 7$
$(7^3)^x = 7^1$
By equating indices
$3x = 1$
$x = 1/3$

(ii) $\log_5 625$

Let $\log_5 625 = x$
$5^x = 625$
$5^x = 5^4$
By equating indices
$x = 4$

Q.5 (a) Find the equation of a circle having its center at the point (2,0) and radius unit.

Ans. - $c\,(h, k) = c\,(2, 0)$ & $r = 1$
Equation of circle
$(x-h)^2 + (y-k)^2 = r^2$
$(x-2)^2 + (y-0)^2 = 1^2$
$x^2 - 4x + 4 + y^2 = 1$
$x^2 + y^2 - 4x + 3 = 0.$

(b) Find the co-ordinates of the centre and radius of a circle whose equation is $x^2+y^2+6x-8y=0$
Ans.- Given equation $x^2+y^2+6x-8y=0$

Comparing above equation with general equation of a circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

$2g = 6,\ 2f = -8,\ c = 0$

$g = 3,\ f = -4,\ c = 0$

But we know that

$h = -g,\ \ k = -f$

$h = -3,\ \ k = 4$

Centre of circle c (h,k) = c(-3,4)

Radius of circle $r = \sqrt{g^2+f^2-c}$

$= \sqrt{3^2+(-4)^2-0}$

$= \sqrt{9+16}$

$= \sqrt{25}$

= 5 unit

Q.6 (a) State any four theorems of limit.

Ans.- Theorems on limit

If $\lim\limits_{x \to a} f(x) = L$ and $\lim\limits_{x \to a} 9(x) = m$ then

(1) $\lim\limits_{x\to a} f(x)\,[f(x) \pm 9(x)] = \lim\limits_{x\to a} \pm \lim\limits_{x\to a} 9(x) = L \pm M$

Thus the limit of sum or difference of the functions is equal to sum or difference of their limits.

(2) If $\lim\limits_{x \to a} k(x) = k \quad \lim\limits_{x \to a} f(x) = K \times L$

Thus the limit of the constant multiplied by a function is equal to constant times of the function

(3) $\lim\limits_{x \to a} [f(x) \times g(x)] \quad \lim\limits_{x \to a} f(x) \times \lim\limits_{x \to a} g(x) = L \times M$

Thus the limit of product of the functions is equal to product of their limits.

(4) $\lim\limits_{x \to a} [f(x) / g(x)] = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} = L/M$

Thus the limit of division of the functions is equal to division of their limits.

(5) $\lim\limits_{x \to a} [f\, g(x)] = f\left[\lim\limits_{x \to a} g(x)\right]$

(b) Evaluate the following limits (Any Two)

(1) $\lim\limits_{x\to 1} (x+1)(x-1)$ (2) $\lim\limits_{x\to 2} \dfrac{x^3-8}{x^5-32}$ (3) $\lim\limits_{x\to 0} \dfrac{\text{Sin}3x}{x}$

(1) $\lim\limits_{x\to 1} (x+1)(x-1)$

$= \lim\limits_{x\to 1} (x+1) \lim\limits_{x\to 1} (x-1)$

$= (1+1)(1-1)$

$= 0$

$\lim\limits_{x\to 2} \dfrac{x^3-2^3/x-2}{x^5-2^5/x-2}$

$= \dfrac{3(2)^2}{5(2)^4}$

$= \dfrac{3}{5x2^2}$

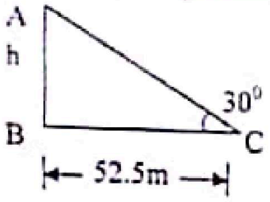
$= \dfrac{3}{}$

$= \lim\limits_{x\to 0} \dfrac{\text{Sin } 3x \text{ X } 3}{3 \text{ X } x}$

$= 1x3$

$= 3$

Q.7 (a) A vertical flagstaff stands on a horizontal place. The angle of elevation of its top was found to be 30^0 from a point at a distance of 52.5m from its foot. Find the height of the flagstaff.



in Δ ABC h = height of the flagstaff

$\tan 30^0 = AB/BC$

$\tan 30^0 = h/BC$

$$\frac{1}{\sqrt{3}} = \frac{h}{52.5}$$

$0.577 = h/52.5$

$h = 0.577 \times 52.5$

$h = 30.29$ m

(b) Enlist any four types of functions with one example each.

Ans. -- (1) Algebraic function or polynomial function

$f(x) = a_0 + a_1x + a_2x^2 + \text{----------} + a_nx^n$ is known as algebraic or polynomial function where a_0, a_1, a_2 ------- a_n are the real numbers and n is an integer.

(2) Rational function :- A ratio of two polynomials forms a rational function

e.g. $f(x) = \dfrac{x^2+3x+5}{x-4}$ if $x \neq 4$

(3) Logarithmic function :- A function defined by $y = f(x) = \log_a x$ is called logarithmic function to the base 'a' where 'a' is any real positive numbers except 1 or A function which contain log terms is called logarithmic function.

e.g. $\log 3^{11}$, $\log 5^5$

(4) Inverse function :- If $y = f(x)$ be the function then $x = f^{-1}(y)$ is called the inverse function of $y = f(x)$

e.g. Let $y = 5x+9$ then $5x = y-9$ therefore $f^{-1}(y) = x = \dfrac{y-9}{5}$

Thus $f^{-1}(y) = x = \dfrac{y-9}{5}$ is inverse of $y=5x+9$

(5) Inverse trigonometric function :- If $y = \sin x$ then $x = \sin^{-1} y$ is called the inverse trigonometric function.

(6) Exponential function :- A function of form $f(x) = a^x$ is called exponential function where $a>0$.

e.g. $y=f(x)=5^x$, $y=f(x)=e^x+3$

(7) Non- algebraic functions :- A function which contains trigonometric logarithmic exponential etc terms called as non=algebraic function.

e.g. $y=f(x)=5+\sin 2x$, $y=e^x$

Q.8 (a) Find the equation of straight line passing through the points whose co-ordinates are (-1,3) and (6,-7).

Ans.:-- Let P (x_1,y_1) = (-1,3) and Q (x_2,y_2)=(6,-7)

i.e. $x_1=-1$, $y_1=3$, $x_2=6$, $y_2=-7$

$$\frac{y-3}{-7-3} = \frac{x-(-1)}{6-(-1)}$$

$$\frac{y-3}{-10} = \frac{x+1}{7}$$

$$7(y-3) = -10(x+1)$$

$$7y-21 = -10x-10$$

$$10x+7y=11$$

(b) Find the co-ordinates of the points which externally divides the line joining the points P(-2,1) and Q(5,7) in the ratio 2:1.

Ans.:- Given P (-2,1) = $P(x_1,y_1)$ and Q(5,7) = $Q(x_2,y_2)$ are the two point, m = 2 n = 1

The co-ordinates of the points which divides the segment PQ externally in the ratio m:n are

$$(X,Y) = \left[\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right]$$

$$= \left[\frac{2x5 - 1x(-2)}{2-1}, \frac{2x7-(1x1)}{2-1}\right]$$

$$= \left[\frac{10+2}{1} \quad \frac{14-1}{1}\right]$$

$$= \left[\frac{12}{1}, \frac{13}{1}\right]$$

$$= (12, 13)$$

Q.9 Differentiate the following functions w r t 'x' (Any Two)

(1) $2x^3 - 3x^2$ (2) $x^2 \sin X$ (3) $(3x^2 - 5x + 7)^{10}$

(1) $2x^3 - 3x^2$

(1) Let $y = 2x^3 - 3x^2$

Differentiate w.r.t. 'X'

$$\frac{dy}{dx} = 2\frac{d}{dx}(X^3) + \chi^2\frac{d}{dx}(2) - \left[3\frac{d}{dx}(x^2) + x^2\frac{d}{dx}(3)\right]$$

$$= 2 \times 3x^2 + x^2(0) - [3 \times 2x + x^2(0)]$$

$$= 6x^2 + 0 - 6x - 0$$

$$= 6x^2 - 6x$$

$$= 6x(x-1)$$

(2) $x^2 \operatorname{Sin} x$

$$y = x^2 . \operatorname{Sin} x$$

Differentiate w.r.t. 'x'

$$\frac{dy}{dx} = x^2\frac{d}{dx}(\sin x) + \sin x\frac{d}{dx}(x^2)$$

$$\frac{dy}{dx} = x^2 . \cos x + 2x \sin x$$

(3) $(3x^2 - 5x + 7)^{10}$

$$y = (3x^2 - 5x + 7)^{10}$$

Differentiate w.r.t. 'x'

$$\frac{dy}{dx} = 10(3x^2 - 5x + 7)^9 \frac{dy}{d\chi}(3x^2 - 5x + 7)$$

$= 10(3x - 3x - 7) \times (6x - 3)$

Q.10 Evaluate the following integrals (Any Four).

(1) $\int \frac{x^3-3x^2+5x-2}{x^3}\,dx$ (2) $\int_a^b e^x\,dx$

(3) $\int \sin 2x\,dx$ (4) $\int_0^2 (3x^2-2x)\,dx$ (5) $\int_0^2 (1/x)\,dx$

(1) $\int \frac{x^3-3x^2+5x-2}{x^3}\,dx$

$= \int dx - 3\int \frac{1}{x}\,dx + 5\int x^{-2}\,dx - 2\int x^{-3}\,dx$

$= x - 3\log x \frac{-5}{x} + \frac{2}{x^2} + c$

(2)

$\int_a^b e^x\,dx$

$\int_a^b e^x\,dx = [e^x]_a^b = e^b - e^a$

(3) $\int \sin 2x\,dx$

$\int \sin 2x\,dx = \frac{-\cos 2x}{2} + c$

(4) $\int_0^2 (3x^2-2x)\,dx$

$\int_0^2 (3x^2-2x)\,dx = 3\int_0^2 x^2\,dx - 2\int_0^2 x\,dx = 3\left[\frac{x^3}{3}\right]^2 - 2$

$= 3\left[\frac{2^3}{3} - \frac{0^3}{3}\right] - 2\left[\frac{2^2}{2} - \frac{0^2}{2}\right]$

$= 3\left[\frac{8}{3}\right] - 2\left[\frac{4}{2}\right]$

$= 8 - 4 = 4$

(5) $\int_0^2 (1/x)\,dx$

$$\int_{0}^{2} \frac{1}{x}\,dx = [\log x]_{1}^{2} = \log 2 - \log 1 = \log 2 - 0 = \log 2$$

SECTION "B"

Q.11 Fill in the blanks

(1) A quadratic equation cannot have more than **Two** roots.

(2) If any two rows or columns of the determinant are identical then the value of the determinant is **Zero.**

(3) $\log_5 5 =$ **1**.

(4) If a point lies in the third quadrant then both the co-ordinates of a point are **Negative.**

(5) $y = \log x + 10$ is **Logarithm** type of function.

(6) The radius of the circle $x^2+y^2=16$ is **4**.

(7) $\int a^2\,dx = \dfrac{ax}{\log x}$

(8) $\lim k =$ **K** where k is constant.

Q.12 State True or False.

(1) The root of quadratic equation $ax^2+bx+c=0$, where $a\neq 0$ will be real and equal if $b^2-4ac=0$. = **True**

(2) The definite integral of a function is unique. = **True**

(3) Simpson's rule is used when even number of ordinates are given. = **False**

(4) $\log(mn) = \log m - \log n$. = **False**

(5) Derivative of constant function is zero. = **True**

(6) The equation of y axis is $x = 0$. = **True**

(7) Point (1, -1) lies in the first quadrant. = **False**

(8) e^x is a logarithmic function. = **False**